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Open/Closed Duality, Unstable D-Branes and Coarse-Grained Closed Strings

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Abstract

At the final stage of unstable D-brane decay in the effective field theory approach, all energy and momentum of the initial state are taken up by two types of fluids, known as string fluid and tachyon matter. In this note, we compare motion of this fluid system to that of macroscopic collection of stretched closed strings and find a precise match at classical level. The string fluid reflects low frequency undulation of the stretched strings while the tachyon matter encodes the average effect of high frequency oscillations turned on those strings. In particular, the combined fluid system has been known to have a reduced speed of light, depending on the composition, and we show that this property is exactly reproduced in classical motion on the closed string side. Finally we illustrate how the tachyon matter may be viewed as an effective degrees of freedom carrying high frequency energy-momentum of Nambu-Goto strings by coarse-graining the dynamics of the latter.

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1 Introduction

The low energy approach [1] to unstable D-branes has proved to be surprisingly rich and successful. Regardless of whether we start with Born-Infeld type action [2, 3, 4, 5, 6, 7] or with that from boundary string field theory (BSFT) [8, 9, 10, 11, 12], it successfully reproduced lower dimensional D-brane as worldvolume solitons with exactly the right tension. Furthermore, starting with Born-Infeld type action of unstable D-brane, the correct low energy effective action of those stable D-branes was obtained [13, 14]. All these happen despite the fact that the open string theory is severely truncated for the low energy approximation. For some reason, the latter remembers quite a lot of what the full string theory is expected to have.

Another interesting aspect of the low energy theory is found in the sector with net electric flux [15, 16]. The gauge field on the unstable D-brane can have conserved electric flux, and these flux lines carry fundamental string charges [17]. When the D-brane decay reaches its final stage, the classical solution of the system is characterized as a two component fluid system [18, 19]. One is the pressureless electric flux lines, dubbed as string fluid [16], while the other is dust-like object known as tachyon matter [20, 21, 22]. Both fluid components behave as perfect fluid, albeit of different dimensionality.

Most general static solution of unstable D-brane decay has been found, and can be described in a simple term [16, 18, 19]: Flux lines should be all collinear but otherwise unrestricted in their distribution. Tachyon matter can be distributed arbitrarily as well, except that its density should be uniform along the flux line direction. This simple state comes with huge degeneracy because transverse (with respect to flux lines) distribution of neither fluid density affects the total energy.*

In the prime motivation for studying this case with net electric flux is a fundamental question on the relationship between open strings and closed strings. The tachyon condensation is supposed to remove all open string degrees of freedom, and the final state must be described in terms of closed strings only. From this viewpoint, the string fluid and tachyon matter must also have a natural interpretation via closed string states. On the other hand, the low energy description of the system is tree-

*A simpler version of such configurations, where both fluids are taken to be uniform, is also realized as a boundary state in string theory [20, 23] and further studied in Ref. [24], although it is very difficult to see dynamics of the final stage from this approach.

level in the open string sense, where closed string is usually not visible. Thus, if we indeed succeed in describing these fluid in terms of closed strings, it begs for further explanations of how the two kinds of strings are related.

The string fluid has been known for some time by now to mimic classical behavior of fundamental string remarkably well [16]. In particular, one could start with a configuration where a finite amount of flux lines are bundled into an infinitesimal tubular shape. Dynamics of such a configuration has been shown to be exactly that of a Nambu-Goto string [16, 25, 26] whose tension is nothing but the total flux. Natural construction of fundamental strings from this is, however, hampered by the degeneracy of the string fluid. No confinement mechanism is known within the classical regime, while a fully quantum scenario has been suggested [15, 17].

More recently, a macroscopic interpretation for the combined system of string fluid and tachyon matter is proposed [27]. Consider a macroscopic number of long fundamental strings lined up along one particular direction, and turn on oscillators along each of those strings in such a way that the oscillator energy density settles down to a uniform value along each and every string. The proposed map is to identify energy of electric flux lines as coming from the "winding" mode part of fundamental strings, while attributing the tachyon matter energy to the oscillator part. Static properties of the two sides were considered and easily shown to be identical [27].

In this note, we will consider the two sides in detail and try to develop more precise dictionary. Since we are trying to map a system of two types of fluid, namely string fluid and tachyon matter, to that of one kind of objects, namely classical closed string, the map cannot be exactly one to one. Rather, there has to be some ambiguity or redundancy in the translation. As we begin detailed discussion below, this point will become clearer.

In section 2, we start with classical motion of a long Nambu-Goto string and discuss stationary solutions and modulations of such stationary solutions. Once a string is endowed with high frequency oscillations along its length, nonlinear nature of Nambu-Goto action sets in, which we will discuss in some detail. We focus on energy-momentum flow along classical strings, and show that on average the flow obeys a modified light-cone in a manner decided by the averaged energy and momentum in the background oscillatory strings. Section 3 will review properties of string fluid and tachyon matter, and compare it against those of long Nambu-Goto string(s). Again

we will concentrate on small fluctuation around stationary solutions, and find that the propagation of disturbances mimics exactly its counterpart in Nambu-Goto strings found in section 2. Section 4 discusses implications of our finding. In section 5, we will make our argument more precise by realizing tachyon matter as a mathematical device that encodes energy and momentum of microscopic oscillations of a string. An outlook is provided in section 6.

2 Classical Motion of Long Nambu-Goto String

We will consider an infinitely long string whose dynamics is govern by the usual Nambu-Goto action. The action is

$$-\frac{1}{2\pi\alpha'}\int d\tau d\sigma\,\sqrt{-\mathrm{Det}\,g}, \tag{2.1}$$

with the induced metric g on the world sheet,

$$g_{ij} = \partial_i X^\mu \partial_j X^\nu \eta_{\mu\nu}. \tag{2.2}$$

Throughout this note we consider flat spacetime only, hence $\eta_{\mu\nu} = \mathrm{diag}(-1, 1, 1, \ldots, 1)$, and furthermore adopt the string unit, $2\pi\alpha' = 1$.

Given that we are interested in long strings, a convenient gauge choice for the worldvolume coordinate is

$$\tau = X^0, \qquad \sigma = X^1, \tag{2.3}$$

upon which we can write the induced metric as

$$g_{ij} = \eta_{ij} + \sum_{I=2}^{D-1} \partial_i X^I \partial_j X^I, \tag{2.4}$$

where ∂X^I $(1 < I \leq D-1)$ represents deviation of the long string from the straight shape.

2.1 Canonical Formulation

Usual method of analyzing the motion of Nambu-Goto string is to introduce intrinsic metric as a Lagrange multiplier field and use the Polyakov action instead. While the latter has obvious advantage in quantization procedure, typical gauge choice there

turns out rather inconvenient for our purpose. We will stick to the Nambu-Goto form and instead go to Hamiltonian formulation. The Hamiltonian density is

$$H = \sqrt{1 + \Pi_I \Pi_I + Y^I Y^I + (\Pi_I Y^I)^2} \tag{2.5}$$

where sum over I is implicit. Π_I is the conjugate momentum of X_I, while we introduced a shorthand notation

$$Y^I \equiv \partial_\sigma X^I. \tag{2.6}$$

Recall that the conserved Noether momentum density on the worldsheet is

$$P \equiv -\frac{\delta L}{\delta \dot{X}^I}\, \partial_\sigma X^I = -\Pi_I\, Y^I. \tag{2.7}$$

From canonical equation of motions, we find

$$\dot{\Pi}_I = \partial_\sigma \left(\frac{Y_I - P\Pi_I}{H} \right), \qquad \dot{Y}^I = \partial_\sigma \left(\frac{\Pi^I - PY^I}{H} \right), \tag{2.8}$$

while the worldsheet energy-momentum conservation may be written as,

$$\dot{H} = -\partial_\sigma P, \qquad -\dot{P} = \partial_\sigma \left(\frac{P^2 - 1}{H} \right). \tag{2.9}$$

2.2 Solutions

We are primarily interested in generic perturbation of string motion around some stable classical solutions. For this let us find typical stationary solution first. An important family of solutions for us are standing wave solutions, i.e. those with no Noether momentum density, $P = -\Pi_I Y_I = 0$. This, together with the energy-momentum conservation, then implies

$$\dot{H} = 0, \qquad \partial_\sigma H = 0. \tag{2.10}$$

We will denote this constant value of H as $\mathcal{E}$ throughout this paper. The simplified equation of motion is

$$\dot{\Pi}_I = \frac{1}{\mathcal{E}}\, \partial_\sigma Y_I, \qquad \dot{Y}^I = \frac{1}{\mathcal{E}}\, \partial_\sigma \Pi^I. \tag{2.11}$$

While the equation may look linear superficially, it is not the case because the constant energy density $\mathcal{E} = (1 + \Pi^2 + Y^2)^{1/2}$ actually depends on specific solutions. The

simplest solution of this type may be found when we consider exciting two transverse directions. For instance,

$$X^2 + iX^3 = \sqrt{\frac{\mathcal{E}^2 - 1}{\mathcal{E}^2\omega^2}}\,\sin\left(\mathcal{E}\omega\sigma\right) e^{\pm i\omega\tau}, \tag{2.12}$$

describes a sinusoidal deviation from the straight string, stabilized by centrifugal force of the rotation in the $X^{2,3}$ plane. This solves Eq. (2.11) and has $P = 0$.

Another important, perhaps more familiar type of solutions involve one-sided propagation of fluctuations. For this, rewrite the Hamiltonian density as

$$H = \sqrt{\left(1 \pm \Pi_I Y^I\right)^2 + \left(\Pi_I \mp Y_I\right)^2}. \tag{2.13}$$

It is not difficult to see that solutions to the first order equation

$$\Pi_I = \pm Y_I, \tag{2.14}$$

reduces the canonical equation of motion to

$$\dot{\Pi}_I = \pm\partial_\sigma \Pi_I, \qquad \dot{Y}_I = \pm\partial_\sigma Y_I, \tag{2.15}$$

which gives simple plane-wave solutions,

$$X^I = \mathrm{Re}\left[C^I e^{i\omega(\sigma\mp\tau)}\right], \tag{2.16}$$

with arbitrary complex numbers, C^I, either moving "up" or "down" along the length of the string at unit speed, depending on the sign. These plane-wave solutions are reminiscent of those used as the basis for quantizing the string in the conformal gauge. Note that our gauge choice is *not* conformal. Although the nonlinearity of Nambu-Goto action is not manifest for these solutions, this apparent linearity is the artifact of the ansatz. The energy density and the momentum density of such solutions are

$$H\Big|_{\Pi=\pm Y} = 1 + \Pi_I\Pi_I, \qquad P\Big|_{\Pi=\pm Y} = \pm\Pi_I\Pi_I. \tag{2.17}$$

We will call these solutions BPS type for an obvious reason.

2.3 Modulating a Standing Wave

As a concrete example of the phenomenon we will see repeatedly, let us take our previous standing wave solution (2.12) and perturb it. The aim of this exercise is two-fold. One is to show that the propagation speed of a perturbation is generically less than unit, perhaps contrary to naive expectation. The other is to visualize two types of perturbations that we will identify as motion of flux lines and also motion of tachyon matter, respectively, in section 3.

For simplicity, we consider motion of string stretched along X^1 and oscillate on the $X^{2,3}$ plane only. Adopting the vector notation $\vec{X} = (X^2, X^3)$, the background standing wave solution (2.12) is

$$\vec{X}_0 = R(\sigma)\hat{n}(\tau), \tag{2.18}$$

where

$$R(\sigma) = \sqrt{\frac{\mathcal{E}^2 - 1}{\mathcal{E}^2\omega^2}}\sin(\mathcal{E}\omega\sigma), \qquad \hat{n}(\tau) = (\cos(\omega\tau), \sin(\omega\tau)). \tag{2.19}$$

Throughout this computation, we will consider the case where ω is large while $\mathcal{E}$ remains finite. Note that in this limit the amplitude of the standing wave scales as $1/\omega$ and can be made arbitrarily small. Thus, when seen from far away, the string will look like a thickened, straight string with mass density larger than usual by the factor $\mathcal{E}$.

One can think of two types of modulations on this oscillating string, "transverse" and "longitudinal," later to be identified with motion of string fluid and motion of tachyon matter, respectively. Of course the reason we can talk about "longitudinal" waves is just that the string that moves about is no longer the bare fundamental string. Rather it is an effective one involving nontrivial solution of fundamental string which broke the translation invariance along the fundamental string.

The former perturbation corresponds to introducing slow undulating motion that involves bending in $X^{2,3}$ plane. If the frequency/wavenumber of this modulation is small enough, the energy involved would be also small even if the amplitude is larger than than of the standing wave. From afar, the resulting motion will appear as the thickened string of mass density $\mathcal{E}$, stretched along X^1, oscillating transversally. In the next section, we will identify this with motion of flux lines.

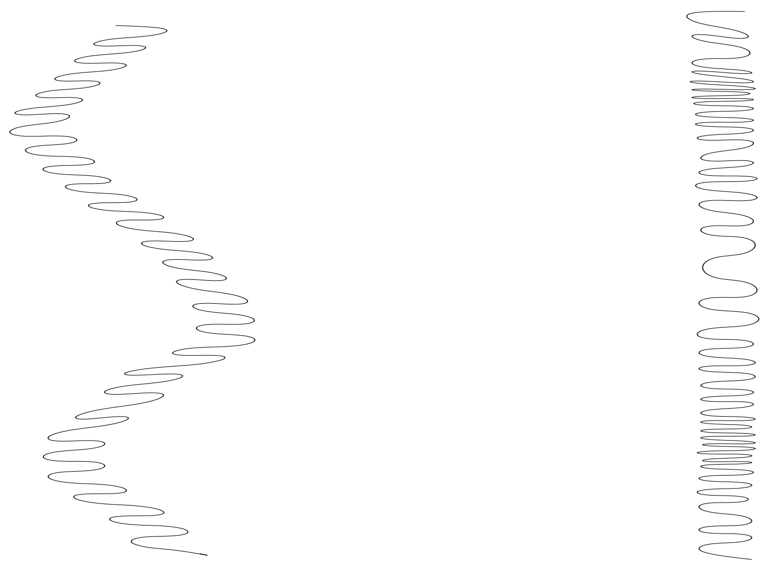

Figure 1: Pictorial representations of two typical perturbations are drawn here. The unperturbed solution would corresponds to high frequency configurations with no modulation. The left side represents "transverse" mode where the fast oscillating string experiences a large scale undulation. The right hand side represents "longitudinal" mode where the high frequency background has its frequency modulated along the length of the string.

Let us consider such "transverse" perturbation first. For this, write the perturbed solution as $\vec{X} = \vec{X}_0 + \delta\vec{X}$ and expand the Nambu-Goto Lagrangian up to second order in $\delta\vec{X}$. For sufficiently large-scale fluctuations $\delta\vec{X}$, we will perform suitable space-time average with respect to the standing wave background to get the effective Lagrangian for $\delta\vec{X}$. The Nambu-Goto Lagrangian is

$$\mathcal{L} = -\sqrt{\Big(1-(\partial_\tau\vec{X})^2\Big)\Big(1+(\partial_\sigma\vec{X})^2\Big) + \Big(\partial_\tau\vec{X}\cdot\partial_\sigma\vec{X}\Big)^2}, \tag{2.20}$$

and the second order expansion looks like

$$\begin{aligned}\mathcal{L} \;\approx\; & -\mathcal{E}(1-\omega^2R^2) + \frac{\mathcal{E}}{2}\delta\vec{X}_\tau^{\,2} - \frac{1}{2\mathcal{E}}\delta\vec{X}'^2 \\ & - \frac{1}{\mathcal{E}(1-\omega^2R^2)}\Big(-2\omega RR'(\hat{\theta}\cdot\delta\vec{X}_\tau)(\hat{n}\cdot\delta\vec{X}') + \frac{1}{2}(\omega R\hat{\theta}\cdot\delta\vec{X}' + R'\hat{n}\cdot\delta\vec{X}_\tau)^2\Big) \\ & + \frac{\mathcal{E}\omega^2R^2}{2(1-\omega^2R^2)}(\hat{\theta}\cdot\delta\vec{X}_\tau)^2 + \frac{(R')^2}{2\mathcal{E}^3(1-\omega^2R^2)}(\hat{n}\cdot\delta\vec{X}')^2 + \ldots,\end{aligned} \tag{2.21}$$

where $\hat{\theta} = (-\sin(\omega\tau), \cos(\omega\tau))$, $R' = \partial_\sigma R$, $\delta\vec{X}_\tau = \partial_\tau \delta\vec{X}$, and $\delta\vec{X}' = \partial_\sigma \delta\vec{X}$. Performing space-time average, for example,

$$\langle \hat{n}^i \hat{n}^j \rangle = \langle \hat{\theta}^i \hat{\theta}^j \rangle = \frac{1}{2}\delta^{ij}, \quad \left\langle \frac{RR'\hat{n}^i\hat{\theta}^j}{1-\omega^2 R^2} \right\rangle = 0, \tag{2.22}$$

we have the effective action for $\delta\vec{X}$,

$$\mathcal{L} \approx -\frac{1+\mathcal{E}^2}{2\mathcal{E}} + C \cdot \Big((\delta\vec{X}_\tau)^2 - \frac{1}{\mathcal{E}^2}(\delta\vec{X}')^2 \Big) + \cdots, \tag{2.23}$$

where the constant C is

$$C = \frac{\mathcal{E}}{2}\Big(1 - \frac{1}{2\mathcal{E}^2} \Big\langle \frac{-1+\mathcal{E}^2 - 2\mathcal{E}^2\omega^2 R^2}{1-\omega^2 R^2} \Big\rangle \Big). \tag{2.24}$$

The above effective action shows that the propagation velocity is reduced to $1/\mathcal{E}$.

We now turn to the "longitudinal" modulation case. Imagine that we slowly change the frequency energy density of the standing wave along X^1 direction. At any given time, the configuration still looks like a thickened string but whose mass density varies along X^1. Of course the variation of energy density will propagate up and down the thickened string, and behaves like an longitudinal wave. For this reason, we will call this second type "longitudinal" perturbation. In next section, we will identify this with motion of tachyon matter.

Such "longitudinal" mode can be realized in many different ways. We choose the following form;

$$\vec{X} = \sqrt{\frac{\mathcal{E}^2 - 1}{\mathcal{E}^2\omega^2}} \sin\Big(\mathcal{E}\omega\sigma + f(\tau,\sigma)\Big)\ \hat{n}(\tau), \tag{2.25}$$

where the modulation mode f is slowly varying. Taking space-time derivatives gives

$$\begin{aligned} \vec{X}_\tau &= \omega R\ \hat{\theta} + f_\tau \frac{R'}{\mathcal{E}\omega}\ \hat{n}, \\ \vec{X}' &= \Big(1 + f'\frac{1}{\mathcal{E}\omega}\Big) R'\ \hat{n}, \end{aligned} \tag{2.26}$$

where $R(\sigma)$ is as before. More precisely, the argument in R is modified by f, but for sufficiently slowly varying f, we can absorb it by a shift in σ for a sizable region of space-time where we are going to average. Its presence thus makes no change in our final averaged effective action. Hence, for our purpose of calculating the effective

action for f, it makes no difference to use

$$\begin{aligned}\vec{X}_\tau &= (\vec{X}_0)_\tau + f_\tau \frac{R'}{\mathcal{E}\omega}\,\hat{n},\\ \vec{X}' &= \vec{X}_0' + f' \frac{R'}{\mathcal{E}\omega}\,\hat{n}.\end{aligned} \tag{2.27}$$

After a straightforward calculation, the second order expansion reads as

$$\mathcal{L} \approx -\mathcal{E}(1-\omega^2R^2) + \frac{-1+\mathcal{E}^2-\mathcal{E}^2\omega^2R^2}{2\mathcal{E}^3\omega^2(1-\omega^2R^2)}\left((f_\tau)^2 - \frac{1}{\mathcal{E}^2}(f')^2\right) + \cdots, \tag{2.28}$$

whose space-time average is

$$\mathcal{L} \approx -\frac{1+\mathcal{E}^2}{2\mathcal{E}} + C'\cdot\left((f_\tau)^2 - \frac{1}{\mathcal{E}^2}(f')^2\right) + \cdots. \tag{2.29}$$

This again reproduces the reduced speed of $1/\mathcal{E}$.

2.4 Perturbing Energy and Momentum

We want to ask the following question: Take a collection of highly oscillatory strings which are roughly straight and collinear, and perturb them by introducing a common low frequency modulation. How does such perturbation propagate along the length of the strings? In the last subsection, we considered this question for two very specific classes of stationary Nambu-Goto string solution and found the answer

$$\text{effective speed of light} = \frac{1}{\mathcal{E}}\quad , \tag{2.30}$$

which is the ratio between energy density of a straight string and that of stretched oscillatory string, per unit distance in spacetime along the string. In this subsection, we will show that this property is not particular to the solutions used, but is much more generic if we consider a large number of nearly coincident strings.

For this, let us concentrate on basic physical properties such as how energy and momentum of the perturbing mode flows. The above result for the standing wave can be understood in a simple and universal manner in this way. For the standing wave solution ($P = 0$), the first order perturbation of energy momentum conservation (2.9) gives,

$$\delta\dot{H} = -\partial_\sigma \delta P, \qquad -\delta\dot{P} = \frac{1}{\mathcal{E}^2}\,\partial_\sigma\delta H. \tag{2.31}$$

Therefore, regardless of details of the perturbation solution, the perturbed energy and momentum obeys a sort of an 1+1 dimensional Klein-Gordon equation;

$$\left[\mathcal{E}^2\partial_\tau^2 - \partial_\sigma^2\right]\delta H = 0, \qquad \left[\mathcal{E}^2\partial_\tau^2 - \partial_\sigma^2\right]\delta P = 0, \tag{2.32}$$

whose propagation velocity is $\pm 1/\mathcal{E}$, inverse of energy density of the standing wave type solution. Remembering the gauge choice for the worldsheet diffeomorphism, $\tau = X^0$ and $\sigma = X^1$, this means that any modulation of the highly oscillatory long string will propagate at reduced speed of light along X^1 direction, as seen by any spacetime observers. This reduced "speed of light" can be rewritten more suggestively as,

$$\frac{1}{\mathcal{E}} = \frac{\text{String Charge per Unit } X^1 \text{ Length}}{\text{Energy per Unit } X^1 \text{ Length}} \leq 1, \tag{2.33}$$

which will be useful later.

Heuristically this reduction may be understood from the fact that per unit X^1 length, the actual length of string is elongated due to the high frequency oscillation. Propagation speed would be unit on the worldsheet when measured with respect to actual induced metric. However, the propagation speed we are discussing above is measured by dX^1/dX^0 which is different from the worldsheet speed.

For more general class of oscillatory solutions, the above derivation will not be applicable directly because the background energy-momentum is not itself constant. On the other hand, if we are primarily interested in slow modulation of energy and momentum, we may as well "average" the background quantities to extract low frequency part only. Then, what do we mean by averaging? One way is to average over time scale $\sim 1/w$ where w is typical high frequency of the oscillatory string. While this might be the simplest conceptually, the averaging procedure we will adopt is actually hinted by the unstable D-brane physics.

Here we are attempting to match the classical low energy system of a unstable D-brane to classical Nambu-Goto strings. In the former system, the initial energy content scales as $1/g_s$, which is arbitrarily large in classical limit. This means that the final state must be composed of a larger number of closed string degrees of freedom, if there is such an interpretation at all. Thus instead of averaging over time along a single string, we may consider a bundle of fundamental strings and average over the

nearby closed strings. In this spirit, Let us replace

$$\begin{aligned} H + \delta H &\rightarrow \mathcal{E} + \delta\mathcal{H}, \\ P + \delta P &\rightarrow \mathcal{P} + \delta\mathcal{P}, \end{aligned} \tag{2.34}$$

where the calligraphic characters denote averaged or low frequency part only. More precise definition of what we mean by this averaging will be discussed in section 5. We will consider only the case where $\mathcal{E}$ and $\mathcal{P}$ are constants in this approximate sense. The BPS plane-wave solution of previous subsection with large ω would be one such example.

The modulation is encoded in $\delta\mathcal{H}$ and $\delta\mathcal{P}$ which we must assume to have only slow dependence on τ and σ. The latter quantities then obey the averaged conservation equation,

$$\delta\dot{\mathcal{H}} = -\partial_\sigma \delta\mathcal{P}, \qquad -\delta\dot{\mathcal{P}} = 2\,\frac{\mathcal{P}}{\mathcal{E}}\,\partial_\sigma\delta\mathcal{P} \;+\; \left(\frac{1-\mathcal{P}^2}{\mathcal{E}^2}\right)\partial_\sigma\delta\mathcal{H}. \tag{2.35}$$

Second order equation for $\delta\mathcal{P}$ is then

$$\left[\mathcal{E}^2\partial_\tau^2 + 2\mathcal{E}\mathcal{P}\partial_\tau\partial_\sigma - (1-\mathcal{P}^2)\partial_\sigma^2\right]\delta\mathcal{P} = 0. \tag{2.36}$$

Extracting the dispersion relation from this, we can solve for relationship between low frequency Ω and low wavenumber K.

$$\delta\mathcal{H}, \delta\mathcal{P} \sim e^{-i\Omega\tau + iK\sigma}. \tag{2.37}$$

The result is

$$\frac{\Omega}{K} = \frac{\mathcal{P}}{\mathcal{E}} \pm \frac{1}{\mathcal{E}}. \tag{2.38}$$

Again the effective "speed of light" as seen by the spacetime observer deviate from unit. Furthermore, modulation speed is asymmetrical; disturbances tends to move faster along the direction of the background momentum $\mathcal{P}$ than along the backward direction. A simple consistency check is to show that the faster of the two speeds does not exceed unit, set by speed of light, 1. This can be see easily using the general formula (2.13),

$$\frac{|\mathcal{P} \pm 1|}{\mathcal{E}} = \frac{|\langle P\rangle \pm 1|}{\langle H\rangle} = \frac{|1 \pm \langle P\rangle|}{\left\langle\sqrt{(1\pm P)^2 + (\Pi_I \pm Y_I)^2}\right\rangle} \;\leq\; \frac{|1 \pm \langle P\rangle|}{\left\langle\sqrt{(1\pm P)^2}\right\rangle} = 1. \tag{2.39}$$

In next section , we will show that this causal properties are exactly matched by the combined system of string fluid and tachyon matter.

3 String Fluid and Tachyon Matter

Tachyon condensation of an unstable D-brane with conserved electric flux on it has been analyzed extensively using an effective field theory of gauge field coupled to tachyon [16, 18, 19, 25].[†]As the tachyon potential approaches to zero, the most convenient description of the system turns out to be canonical Hamiltonian formalism. In Ref. [19], all static with no net momentum solutions have been characterized as straight conserved flux lines with tachyon matter distributed uniformly along the flux direction. Transverse variations of both flux density and tachyon matter are unconstrained, implying huge degeneracy of the solutions.

In this section, we perform similar analysis to the previous section of looking at small perturbation spectrum for stationary states of string fluid with tachyon matter. As in the Nambu-Goto string case, we include the situation where the system has a constant momentum density whose direction is along the flux line. The existence of this type of configuration is obvious by boosting the solution with no momentum along the string direction. In fact, the dispersion relation of perturbations for this system may be obtained by simple Lorentz transformation. We will check the consistency our results with Lorentz symmetry.

We will then compare the results of this section with those in the previous section for Nambu-Goto string, and find intriguing similarity between them. Pushing forward this observation, we explore a possibility of identifying the system of flux lines to a macroscopic collection of fundamental strings, with the tachyon matter interpreted as an effective degree of freedom describing small fast oscillations of fundamental strings.

3.1 Stationary Solutions

We start from the effective action for an unstable Dp-brane, including tachyon as a dynamical degree of freedom

$$-\int d^{p+1}x\; V(T)\sqrt{-\det(\eta_{\mu\nu}+F_{\mu\nu}+\partial_\mu T\partial_\nu T)}. \tag{3.1}$$

[†]A fluid-like behavior of Born-Infeld action in the string coupling limit, i.e., in the limit of vanishing tension, was previously observed in Ref. [28, 29, 30].

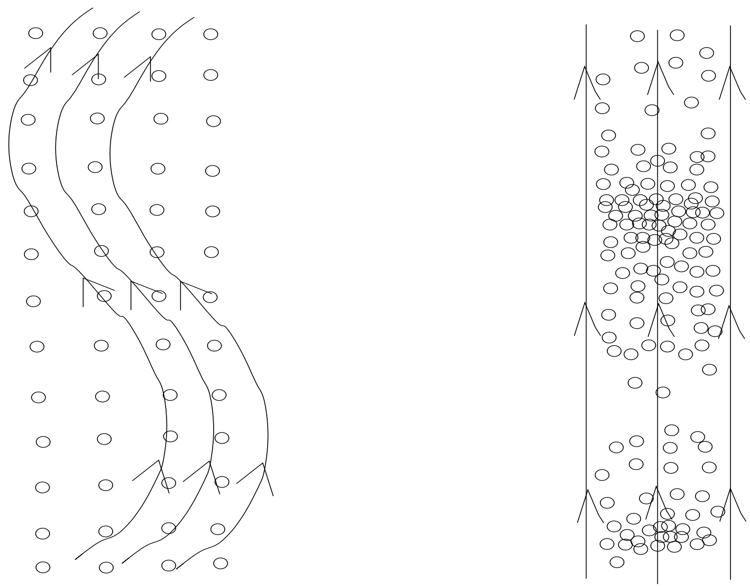

Figure 2: Perturbation associated with the two types of fluids are drawn here. The small circles represent tachyon matter, while the lines are string fluid. In the background of static solution, two allowed perturbations are transverse motions of string fluid itself (left) and arbitrary motion of tachyon matter (right). The two map to closed string picture, respectively, as "transverse" and "longitudinal" modulation of collection of fundamental strings with random high frequency modes turned on.

Upon tachyon condensation $V \to 0$, the system is described in canonical formalism by the Hamiltonian [16]

$$\mathcal{H} = \sqrt{\pi^i \pi^i + \pi^T \pi^T + (\partial_i T \pi^i)^2 + \mathcal{P}_i \mathcal{P}_i} \quad , \tag{3.2}$$

where π^i, π^T are conjugate momenta of gauge fields and tachyon, and $\mathcal{P}_i = -F_{ij}\pi^j - \partial_i T \pi^T$ are the momentum densities. For future convenience, let us recapitulate the energy-momentum tensor of the theory in tachyon condensation limit [18],

$$\begin{aligned} T_{00} &= \mathcal{H}, \\ T_{0i} &= -\mathcal{P}_i, \\ T_{ij} &= \frac{-\pi^i \pi^j + \mathcal{P}_i \mathcal{P}_j}{\mathcal{H}}. \end{aligned} \tag{3.3}$$

It is rather straightforward to derive the equations of motion from the above Hamiltonian. However, there is a more intuitive description of the system in terms of fluid-like variables [16], whose existence reflects an intrinsic property of the system as some kind of string fluid. Define (Indices i, j, k run from 1 to p)

$$\begin{aligned} n^k = \frac{\pi^k}{\mathcal{H}} \qquad &, \qquad n^T = \frac{\pi^T}{\mathcal{H}}, \\ v^k = \frac{\mathcal{P}_k}{\mathcal{H}} \qquad &, \qquad v^T = \frac{\partial_i T \pi^i}{\mathcal{H}}, \end{aligned} \tag{3.4}$$

Then, the half of canonical equations of motion combined with energy-momentum conservation gives

$$\begin{aligned} \partial_0 n^k + v^i \partial_i n^k &= n^i \partial_i v^k, \\ \partial_0 n^T + v^i \partial_i n^T &= n^i \partial_i v^T, \\ \partial_0 v^k + v^i \partial_i v^k &= n^i \partial_i n^k, \\ \partial_0 v^T + v^i \partial_i v^T &= n^i \partial_i n^T, \end{aligned} \tag{3.5}$$

together with constraints,

$$n^i n^i + n^T n^T + v^i v^i + v^T v^T = 1, \qquad n^i v^i + n^T v^T = 0, \tag{3.6}$$

and an integrability condition that F_{ij} and $\partial_i T$, which can be solved from the other half of canonical equations of motion with the above fluid background, should satisfy

$$\begin{aligned} v^i &= -F_{ij} n^j - n^T \partial_i T, \\ v^T &= \partial_i T n^i. \end{aligned} \tag{3.7}$$

For the case of static ($\partial_0 = 0$, $\mathcal{P}_i = 0$) configurations, it is not difficult to find the general form of solutions [19]. The flux density with specified constant direction, say π^1 (or n^1), and the tachyon matter π^T (or n^T) are the only non-vanishing quantities in this case. They may have arbitrary transverse profiles to x^1, but restrained to be constant along x^1. Complete characterization of these solutions up to overall rotation is given by the following three nontrivial quantities,

$$\begin{aligned} &\mathcal{H} = \mathcal{H}(x^2, \ldots, x^p), \\ &n^1 = n^1(x^2, \ldots, x^p), \qquad n^T = n^T(x^2, \ldots, x^p), \end{aligned} \tag{3.8}$$

while all others vanish. Magnetic fields F_{ij} and tachyon gradient $\partial_i T$ are absent in these solutions.

We are interested in slightly more general situation than this, however. We have in mind configurations in which there is a net momentum density along x^1, whose existence may easily be inferred by boosting the static case. Once we turn on $\mathcal{P}_1$ (or v^1), Eq.(3.6) tells us that we need to turn on v^T simultaneously, indicating a constant tachyon gradient $\partial_1 T$. Because we boosted the system without magnetic fields along the direction of electric field, we still do not have any magnetic fields, $F_{ij} = 0$, henceforth gauge fields do not contribute to the momentum density;

$$\mathcal{P}_1 = -F_{1j}\pi^j - \partial_1 T \pi^T = -\partial_1 T \pi^T. \tag{3.9}$$

The Hamiltonian (3.2) becomes

$$\begin{aligned} \mathcal{H} &= \sqrt{(\pi^1)^2 + (\pi^T)^2 + (\partial_1 T \pi^1)^2 + (\partial_1 T \pi^T)^2} \\ &= \sqrt{(\pi^1)^2 + (\pi^T)^2} \cdot \sqrt{1 + (\partial_1 T)^2}, \end{aligned} \tag{3.10}$$

and the constraints (3.6) are easily satisfied. Thus most general stationary solution is characterized by

$$\begin{aligned} &\mathcal{H} = \mathcal{H}(x^2, \ldots, x^p), \\ &n^1 = n^1(x^2, \ldots, x^p), \qquad n^T = n^T(x^2, \ldots, x^p), \\ &v^1 = v^1(x^2, \ldots, x^p), \qquad v^T = -n^1 v^1 / n^T. \end{aligned} \tag{3.11}$$

This latter class of solution cannot be extended to everywhere because the gradient of T will eventually introduce a region where $V \neq 0$, and possibly domain walls, leading us out of the region of validity for the above equation of motions. However, we are mainly interested in general motion of string fluid in the vacuum, and local solutions are enough.

3.2 Perturbation

A linear perturbation to the fluid equation (3.5) in the background of the previous subsection reads

$$\partial_0 \delta n^\alpha + v^1 \partial_1 \delta n^\alpha - n^1 \partial_1 \delta v^\alpha = 0, \tag{3.12}$$

$$\partial_0 \delta v^\alpha + v^1 \partial_1 \delta v^\alpha - n^1 \partial_1 \delta n^\alpha = 0, \tag{3.13}$$

where α runs not only from 1 to p, but also the tachyon direction T. Remember that the background quantities, like n^1 and v^1, are constant along x^1 direction. Taking time derivative of the first equation and replacing δv^α in favor of δn^α by judicious use of the two equations produce

$$\partial_0^2 \delta n^\alpha + 2v^1 \partial_0 \partial_1 \delta n^\alpha + ((v^1)^2 - (n^1)^2)\partial_1^2 \delta n^\alpha = 0, \tag{3.14}$$

and the same equation for δv^α. The resulting dispersion relation between frequency Ω and wave number K

$$\delta n, \delta v \sim e^{-i\Omega t + iKx^1}, \tag{3.15}$$

is

$$\frac{\Omega}{K} = v^1 \pm n^1 = \frac{\mathcal{P}_1}{\mathcal{H}} \pm \frac{\pi^1}{\mathcal{H}}. \tag{3.16}$$

We now notice a striking similarity of the above dispersion relation to that of Nambu-Goto strings, Eq.(2.38). In fact, a bundle of flux lines with unit string charge has a line-energy density $\mathcal{E} = \mathcal{H}/\pi^1$ and a line-momentum density $\mathcal{P} = \mathcal{P}_1/\pi^1$, in terms of which the two formulae match precisely. In the next section, we further elaborate on possible relations between Nambu-Goto strings and string fluid with tachyon matter.

It is not particularly obvious how such a formula for velocity is consistent with laws of relativity. As in the case of fundamental string, we can ask whether the modified propagation speed never exceeds unit. This follows from the constraint on n and v

$$0 \leq (v^1 \pm n^1)^2 = (1 - n_T^2 + v_T^2) \mp n_T v_T = 1 - (n_T \pm v_T)^2 \leq 1. \tag{3.17}$$

As a further consistency check let us show that the dispersion relation, (3.16), transforms under Lorentz boost as it should. Ω/K represents the velocity of small fluctuation signals, and we know how velocities should add under boost transformations. Under a Lorentz boost along direction x^1 with velocity β, a velocity u along x^1 transform as

$$u' = \frac{u - \beta}{1 - \beta\, u}. \tag{3.18}$$

Suppose a system with constant $\mathcal{H}$ and $\mathcal{P}_1$, in which perturbation signals are claimed to propagate with the velocity $u = \frac{\mathcal{P}_1 \pm \pi^1}{\mathcal{H}}$. Now consider another reference frame which is moving along x^1 direction with a velocity β. Recalling that $T_{11} = \frac{-(\pi^1)^2 + (\mathcal{P}_1)^2}{\mathcal{H}}$

is present in the system, the energy and momentum density of the moving system are given by

$$
\begin{aligned}
\mathcal{H}' &= \gamma^2 T_{00} + 2\gamma^2\beta T_{01} + \gamma^2\beta^2 T_{11} \\
&= \gamma^2\mathcal{H} - 2\gamma^2\beta\mathcal{P}_1 + \gamma^2\beta^2\frac{-(\pi^1)^2 + (\mathcal{P}_1)^2}{\mathcal{H}} \quad , \qquad (3.19)\\
\mathcal{P}_1' &= -\gamma^2\beta T_{00} - \gamma^2(1+\beta^2)T_{01} - \gamma^2\beta T_{11} \\
&= -\gamma^2\beta\mathcal{H} + \gamma^2(1+\beta^2)\mathcal{P}_1 - \gamma^2\beta\frac{-(\pi^1)^2 + (\mathcal{P}_1)^2}{\mathcal{H}} \quad , \qquad (3.20)
\end{aligned}
$$

where $\gamma^2 = 1/(1-\beta^2)$. For this particular class of Lorentz boost, it turns out that $\pi'^1 = \pi^1$. With a little bit of algebra, then, it is straightforward to show that

$$
u' = \frac{\mathcal{P}_1'}{\mathcal{H}'} \pm \frac{\pi'^1}{\mathcal{H}'} = \frac{\left(\frac{\mathcal{P}_1}{\mathcal{H}} \pm \frac{\pi^1}{\mathcal{H}}\right) - \beta}{1 - \beta\left(\frac{\mathcal{P}_1}{\mathcal{H}} \pm \frac{\pi^1}{\mathcal{H}}\right)} = \frac{u-\beta}{1-\beta\, u} \quad . \qquad (3.21)
$$

just as a velocity should.

4 Closed Strings Picture: Classical Limit

In section 2, we have seen that small fluctuations on top of oscillatory Nambu-Goto strings travel with the universal velocity, $u = \mathcal{P}/\mathcal{E} \pm 1/\mathcal{E}$, without any regard to the character of perturbations, as long as they are slowly varying with large wavelengths compared to the background oscillation. An observer with her low-energy, large-scale eyes may be able to see large-scale transverse fluctuations, but wouldn't have ability to tell microscopic details of oscillatory nature of the string, except its energy and momentum. Hence, a large-scale observer may describe the system in terms of transverse string distortions plus some new effective degree of freedom that carries energy-momentum of averaged microscopic modes.

On the other hand, analysis in section 3 revealed a compelling interpretation of flux lines coupled to tachyon matter as a macroscopic collection of oscillatory strings. Perturbations of the system consist of transverse distortions of flux lines plus additional mode of tachyon matter fluctuation. Their propagation velocity is again universal, and in fact, it is identical to that of Nambu-Goto string when expressed in terms of properly normalized energy-momentum line-density for unit string charge. We are thus naturally led to relate tachyon matter perturbation in string fluid side

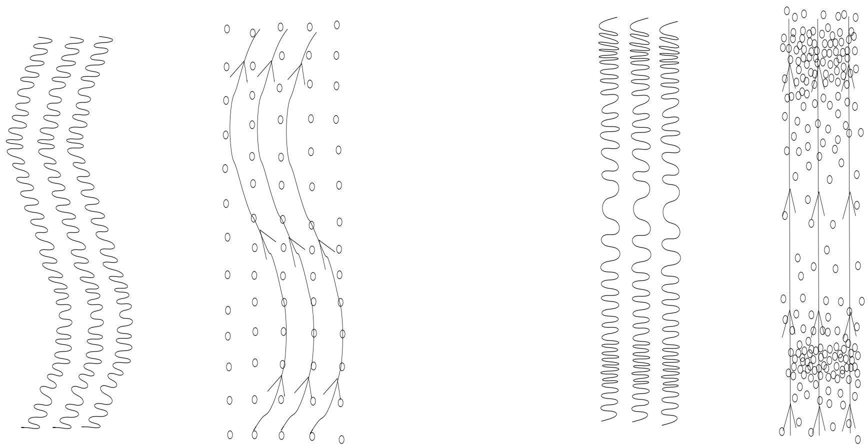

Figure 3: Identifications between modes of Nambu-Goto strings (lefthand side in each pair) and fluctuations of electric flux lines with tachyon matter (right-hand side in each pair). Lines with arrows represent electric flux while circles represent tachyon matter.

with the effective scalar degree of freedom on Nambu-Goto string that encodes microscopic longitudinal fluctuations. This complete matching of classical stationary states and perturbation thereof indicates that this additional degree of freedom is effectively represented by the tachyon matter on the open string side.

With this, it is obvious that the macroscopic interpretation of the string fluid/tachyon matter hybrid is consistent at least at the level of classical limit. What we have gathered here are compelling evidences that the combined system of string fluid and tachyon matter in the open string side actually describes classical motion of stretched fundamental strings.

The interpretation of string fluid as macroscopic snapshot of collection of many fundamental string was quite natural from its classical dynamics, and in fact was already suggested in Ref. [16]. On the other hand, interpretation of tachyon matter is less straightforward. In this note, we are advocating the viewpoint taken in Ref. [27], where the energy associated with tachyon matter is attributed to high frequency oscillations of the stretched fundamental strings themselves.

This identification was suggested by first considering static and stationary states of two sides [27], and here we strengthened the match by further comparing the behavior

of small fluctuations. The motion of string fluid is mapped to large scale motion of stretched strings, while motion of tachyon matter is shown to encode energy-momentum content of high frequency oscillations on the same strings. The system of string fluid and tachyon matter is then interpreted as a coarse-grained view of macroscopic number of stretched fundamental strings. We find these identifications between open string side and closed string side quite compelling.

5 Tachyon Matter from Coarse Graining of Fundamental Strings

In the previous sections, the matching of fluctuation dynamics between a macroscopic collection of Nambu-Goto strings and the system of string fluid with tachyon matter was, in itself, a compelling evidence of our claim. In this section, we try to make a further step illustrating how tachyon matter degrees of freedom may arise on the Nambu-Goto side via some coarse graining procedure.

While this should be possible in general setting, we will illustrate it by matching variables with the case of unstable D1 decay. Start with the unstable D1 action

$$-\int dx^2\, V(T)\sqrt{-\mathrm{Det}\left(\eta_{jk}+\partial_j Z^I\partial_k Z^I+F_{jk}+\partial_j T\partial_k T\right)}\quad , \tag{5.1}$$

with transverse scalars Z^I. Because D1 is also 1+1 dimensional object, it is sensible to use the same index $I=2,3,\ldots,D-1$ as in Nambu-Goto action of fundamental string.

Let us put $\mathcal{N}$ unit of fundamental string charge, that is, electric flux $\pi_1=\mathcal{N}$ on the unstable D1, and let the combined system decay. The final form of Hamiltonian is

$$\mathcal{H}=\sqrt{\mathcal{N}^2+\pi_I^2+\pi_T^2+\mathcal{P}_1^2+\mathcal{P}_I{}^2+\mathcal{P}_T^2}\quad , \tag{5.2}$$

where π_I and π_T are conjugate momenta of transverse scalars Z^I and tachyon T, respectively. The other three quantities are

$$\begin{aligned}\mathcal{P}_1 &= -\partial_1 Z^I\pi_I-\partial_1 T\pi_T,\\ \mathcal{P}_I &= \mathcal{N}\partial_1 Z^I,\\ \mathcal{P}_T &= \mathcal{N}\partial_1 T.\end{aligned} \tag{5.3}$$

Equations of motions for this system look simpler when we again introduce fluid variables,

$$(n_s) = \left(\frac{\mathcal{N}}{\mathcal{H}}, \frac{\pi_I}{\mathcal{H}}, \frac{\pi_T}{\mathcal{H}}\right), \qquad (v_s) = \left(\frac{\mathcal{P}_1}{\mathcal{H}}, \frac{\mathcal{P}_I}{\mathcal{H}}, \frac{\mathcal{P}_T}{\mathcal{H}}\right), \tag{5.4}$$

with s running over $1, I = 2, \ldots, D-1$ and T. The complete low energy dynamics is encoded in

$$\dot{n}_s + v^1 \partial_1 n_s = n^1 \partial_1 v_s, \qquad \dot{v}_s + v^1 \partial_1 v_s = n^1 \partial_1 n_s. \tag{5.5}$$

Note that $s = 1$ part of these equations is nothing but the energy-momentum conservation, once we note that Gauss law is readily incorporated in $\pi_1 = \mathcal{N}$.

In the weak coupling limit, $g_s \to 0$, the initial amount of energy on unstable D1-brane is large $\sim 1/g_s$. This energy must be shared by π_1, π_T, etc, which means that typical value of $\mathcal{N} \sim 1/g_s$ is also large. On the other hand, $\mathcal{N}$ is the fundamental string charge and the corresponding closed string picture should involve many nearly aligned fundamental string moving under Nambu-Goto dynamics. We wish to recover this form of low energy dynamics, in particular the part associated with tachyon matter, by coarse-graining the Nambu-Goto dynamics of many long strings.

To compare with the motion of fundamental string, consider $\mathcal{N}$ infinitely stretched Nambu-Goto strings, whose Hamiltonian is the simple sum of $\mathcal{N}$ separate Nambu-Goto strings,

$$H_{\mathcal{N}} = \sum_{a=1}^{\mathcal{N}} H^{(a)}. \tag{5.6}$$

We labelled the strings by the index (a). Interestingly, each Nambu-Goto dynamics admits variables which behave similarly as the fluid variables in the string fluid system,

$$N^1 \equiv \frac{1}{H}\ , \quad N^I \equiv \frac{Y^I}{H}\ , \quad V^1 \equiv \frac{P}{H}\ , \quad V^I \equiv \frac{\Pi_I}{H}\ , \tag{5.7}$$

where the superscript in N^1 and V^1 means world-sheet σ direction, while we suppressed (a) index since every single string obeys the same equation of motion, decoupled from each other: They satisfy ($m = (1, I)$)

$$\begin{aligned} \partial_\tau N^m + V^1 \partial_\sigma N^m &= N^1 \partial_\sigma V^m, \\ \partial_\tau V^m + V^1 \partial_\sigma V^m &= N^1 \partial_\sigma N^m. \end{aligned} \tag{5.8}$$

By definition, they are constrained by

$$
\begin{aligned}
(N^1)^2 + (N^I)^2 + (V^1)^2 + (V^I)^2 &= 1, \\
N^1 V^1 + N^I V^I &= 0.
\end{aligned} \tag{5.9}
$$

Because we can solve N^1 and V^1 from the above constraints, the independent degrees of freedom are just transverse fluctuations, as should be obvious from the original Hamiltonian.

We now consider the situation where these $\mathcal{N}(\gg 1)$ fundamental strings are roughly coincident, and try to derive a coarse-grained system of equations which involve macroscopic motion of the bundle of strings. High frequency oscillations of individual strings are not something we can see in the coarse grained dynamics, and thus will be deemed to be completely random. For instance, we could imagine a collection of BPS type solutions of section 2 with arbitrary initial conditions and arbitrary high frequency ω for each string. In such a collection, half of strings will have $\omega \gg 1$ and the other half, $-\omega \gg 1$. In contrast, we are interested in overall slow motion of the bundle, so low frequency motion of strings are taken to be identical.

To illustrate the averaging procedure, let us take the equation (5.8) and express it in a Fourier transformed basis by swapping σ in favor of its conjugate variable,

$$
N^m = \frac{1}{2\pi} \int dp \, \hat{N}^m(p;\tau)\, e^{ip\sigma}, \qquad V^m = \frac{1}{2\pi} \int dp \, \hat{V}^m(p;\tau)\, e^{ip\sigma}, \tag{5.10}
$$

we find equations like

$$
\partial_\tau \hat{N}^m(k) + i \int \left(\hat{V}^1(k-p)\hat{N}^m(p) - \hat{N}^1(k-p)\hat{V}^m(p) \right) p \, dp = 0. \tag{5.11}
$$

We will consider only part of this equation and restrict our attention to small k. Imagine that the motion consists of two separate components; one with small $p \sim k$ corresponding to slow modulations under which $\mathcal{N}$ strings move together and the other with very large $p \to \omega$ with $|\omega| \gg |k|$ under which each string moves independently. Then we may split the integral in two parts, according to the range of p-integration,

$$
\begin{aligned}
&\partial_\tau \hat{N}^m(k) + i \int_{small} \left(\hat{V}^1(k-p)\hat{N}^m(p) - \hat{N}^1(k-p)\hat{V}^m(p) \right) p \, dp \\
&= -i \int_{large} \left(\hat{V}^1(k-\omega)\hat{N}^m(\omega) - \hat{N}^1(k-\omega)\hat{V}^m(\omega) \right) \omega \, d\omega \quad .
\end{aligned} \tag{5.12}
$$

Modes that can contribute to the right hand side are the random high frequency modes.

It is our belief that with sufficient randomness on high frequency part thrown in, the right hand side of this equation will be negligible upon the averaging over many nearly coincident strings. For individual strings, each term in this equation must be roughly of the same size. For a random collection of highly oscillatory strings with arbitrary phase, it is likely that the averaging procedure will introduce cancellations among nearby strings. It is important to note that we are considering the larger scale motion of these strings to be roughly the same. Only those variables appearing on the right hand side are random.

For the sake of argument, we will proceed with this assumption. Then, one is left with low frequency modes only. Performing inverse Fourier transformation, we define

$$\tilde{n}^m = \frac{1}{2\pi}\int_{small} dk\ \hat{N}^m(k;\tau)e^{ik\sigma}, \qquad \tilde{v}^m = \frac{1}{2\pi}\int_{small} dk\ \hat{V}^m(k;\tau)e^{ik\sigma}. \tag{5.13}$$

This sort of operation is what we mean by "averaging" or "coarse-graining" the fundamental strings.

The averaged equation of motion for these slow variables are simply (5.8) except everything is replaced by tilde variables,

$$\begin{aligned} \partial_0\tilde{n}^m + \tilde{v}^1\partial_1\tilde{n}^m &= \tilde{n}^1\partial_1\tilde{v}^m, \\ \partial_0\tilde{v}^m + \tilde{v}^1\partial_1\tilde{v}^m &= \tilde{n}^1\partial_1\tilde{n}^m. \end{aligned} \tag{5.14}$$

Now, the crucial difference from the original equation of motion is that our tilde variables need not saturate the constraints (5.9). This is understandable because transverse fluctuations do not saturate all degrees of freedom of our low energy theory, and the energy-momentum have additional independent degree of freedom. Interestingly, we can take an analogy at this point to the string fluid coupled to tachyon matter, and introduce fictitious 'tachyonic' variables $\tilde{n}^T$ and $\tilde{v}^T$ to saturate the constraints (5.9) or the energy-momentum of the system,

$$\begin{aligned} (\tilde{n}^T)^2 + (\tilde{v}^T)^2 &= 1 - (\tilde{n}^1)^2 - (\tilde{n}^I)^2 - (\tilde{v}^1)^2 - (\tilde{v}^I)^2, \\ \tilde{n}^T\tilde{v}^T &= -\tilde{n}^1\tilde{v}^1 - \tilde{n}^I\tilde{v}^I, \end{aligned} \tag{5.15}$$

which may be solved

$$\tilde{n}^T = \frac{1}{2}\left\{\sqrt{1-(\tilde{n}^I+\tilde{v}^I)^2-(\tilde{n}^1+\tilde{v}^1)^2} + \sqrt{1-(\tilde{n}^I-\tilde{v}^I)^2-(\tilde{n}^1-\tilde{v}^1)^2}\right\},$$

$$\tilde{v}^T = \frac{1}{2}\left\{\sqrt{1-(\tilde{n}^I+\tilde{v}^I)^2-(\tilde{n}^1+\tilde{v}^1)^2}-\sqrt{1-(\tilde{n}^I-\tilde{v}^I)^2-(\tilde{n}^1-\tilde{v}^1)^2}\right\} \tag{5.16}$$

An intriguing fact is that they satisfy the exactly same form of time-evolution equations as other $\tilde{n}^I$'s and $\tilde{v}^I$'s do,

$$\begin{aligned} \partial_0\tilde{n}^T + \tilde{v}^1\partial_1\tilde{n}^T &= \tilde{n}^1\partial_1\tilde{v}^T, \\ \partial_0\tilde{v}^T + \tilde{v}^1\partial_1\tilde{v}^T &= \tilde{n}^1\partial_1\tilde{n}^T. \end{aligned} \tag{5.17}$$

which can be checked straightforwardly using (5.14).

The above set of low-energy effective equations of motion for Nambu-Goto string looks precisely same as those for string fluid with tachyon matter, (5.5). Although this already provides a compelling hint at the identity of tachyon matter as an effective degree of freedom representing small oscillations of fundamental strings, we may go further and show the correspondence more explicitly. Guided by the definitions of the fluid variables in the string fluid side, let us define $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{P}}_1$ by

$$\tilde{n}^1 = \left\langle\frac{\mathcal{N}}{H_{\mathcal{N}}}\right\rangle \equiv \frac{\mathcal{N}}{\tilde{\mathcal{H}}}, \quad \tilde{v}^1 = \left\langle\frac{P_{\mathcal{N}}}{H_{\mathcal{N}}}\right\rangle \equiv \frac{\tilde{\mathcal{P}}_1}{\tilde{\mathcal{H}}}. \tag{5.18}$$

It follows that

$$\tilde{n}^I = \tilde{Y}^I\left\langle\frac{\mathcal{N}}{H_{\mathcal{N}}}\right\rangle = \frac{\mathcal{N}\tilde{Y}^I}{\tilde{\mathcal{H}}}, \quad \tilde{v}^I = \tilde{\Pi}_I\left\langle\frac{\mathcal{N}}{H_{\mathcal{N}}}\right\rangle = \frac{\mathcal{N}\tilde{\Pi}_I}{\tilde{\mathcal{H}}}, \tag{5.19}$$

where $\tilde{Y}^I$ and $\tilde{\Pi}_I$ are slowly varying fluctuations. Furthermore, we also introduce an effective field $\tilde{T}$ and its conjugate $\tilde{\pi}_T$, which encode energy-momentum of small oscillations,

$$\tilde{n}^T \equiv \frac{\mathcal{N}\partial_1\tilde{T}}{\tilde{\mathcal{H}}}, \quad \tilde{v}^T \equiv \frac{\tilde{\pi}_T}{\tilde{\mathcal{H}}}. \tag{5.20}$$

Now the constraints (5.15) read as

$$\tilde{\mathcal{H}} = \sqrt{(\mathcal{N})^2+(\mathcal{N}\tilde{Y}^I)^2+(\tilde{\pi}_T)^2+(\tilde{\mathcal{P}}_1)^2+(\mathcal{N}\tilde{\Pi}_I)^2+(\mathcal{N}\partial_1\tilde{T})^2}\quad, \tag{5.21}$$

and

$$\tilde{\mathcal{P}}_1 + (\tilde{\Pi}_I)(\mathcal{N}\tilde{Y}^I) + (\partial_1\tilde{T})(\tilde{\pi}_T) = 0. \tag{5.22}$$

Noting the striking similarity of the above to the Hamiltonian (5.2) and momenta (5.3) of the string fluid system, we are ready to describe the mapping between the

two. Firstly, it is geometrically clear that $\mathcal{N}\tilde{Y}^I = \pi_1\tilde{Y}^I$ should be mapped to $\mathcal{P}_I$, observing that $\tilde{Y}^I = \partial_1 Z^I$. It is also obvious to identify the conjugate momenta π_I with $\mathcal{N}\tilde{\Pi}_I$. Under these and $\tilde{T} \to T$, the Hamiltonian and momenta match precisely. We stress that this matching is not a mere artifact of our definitions of $\tilde{\mathcal{H}}$ and $\tilde{\mathcal{P}}_1$, because the fluid equations, which are identical on both systems, are non trivial consequences of this Hamiltonian and the relevant canonical structure.

This implies that we may consider $\tilde{T} \to T$ and $\tilde{\pi}_T \to \pi_T$ be dynamical variables, instead of being functions of $\tilde{Y}^I$ and $\tilde{\Pi}_I$. Once we assign the canonical Poisson bracket,

$$\{T, \pi_T\}_{P.B.} = \delta, \tag{5.23}$$

also consider the Hamiltonian $\tilde{\mathcal{H}}$ as a function of $(D-2)+1$ pairs of conjugate variables, $\{Z^I, \pi_I; T, \pi_T\}$, the evolution equation for the $\tilde{n}^T$, $\tilde{v}_T$ as well as those for $\tilde{H}$, $\tilde{\mathcal{P}}_1$, $\tilde{n}^I$, and $\tilde{v}^I$ are all correctly reproduced. Poisson bracket for π_I and Z^I may look as if they conflicted with those of $\tilde{\Pi}_I$ and $\tilde{Y}^I$, due to the factor of $\mathcal{N}$. Upon closer inspection, we must realize that the identification is

$$\pi_I = \sum_a \Pi_I^{(a)}, \qquad Z^I = \frac{1}{\mathcal{N}} \sum_a \tilde{Y}^I_{(a)}, \tag{5.24}$$

where the sum is over the $\mathcal{N}$ Nambu-Goto strings. This produces the expected Poisson bracket for the averaged variables. Thus, we managed to construct an effective scalar field T on a coarse-grained Nambu-Goto string, which captures the energy-momentum content of microscopic part of the string oscillation.

6 Beyond Classical Limit

We have provided several arguments supporting the claim that string fluid with tachyon matter describes a macroscopic bundle of fundamental strings. What this means in the larger issue of fully quantum closed strings in the open string tachyon condensation remains to be seen. In a way, the above example of macroscopic interpretation in fully classical limit is itself quite surprising and may lead to hitherto unknown relationship between closed strings and open strings.

At the same time, we are again tempted to ask questions about whether and how some part of this story with classical story is lifted to the fully quantum level. Recent

progresses found a fertile ground for asking such a fully quantum mechanical question, namely old matrix models reinterpreted as a theory of an open string tachyon [31, 32, 33, 34, 35, 36, 37, 38]. In this simplified setting of two spacetime dimensions, closed strings emerge as a collective excitation of the open string tachyon theory in the fully non perturbative definition of the latter.

One possible scenario of how fundamental strings would show up in the quantum open string theory in general dimensions was given some time ago [15, 17]. It remains to be seen whether one could find a common ground among the fully quantum mechanical case (but limited to 2-spacetime dimensions), a low energy viewpoint of the present note, and the somewhat pictorial but fully quantum mechanical scenario.

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